Course: Master in International Finance

Subject: Investment

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Assignment #1

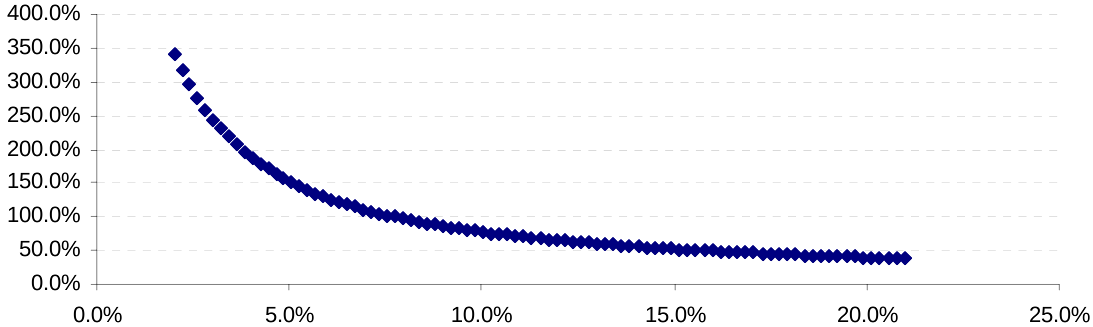
Walt Disney Case Study

A.

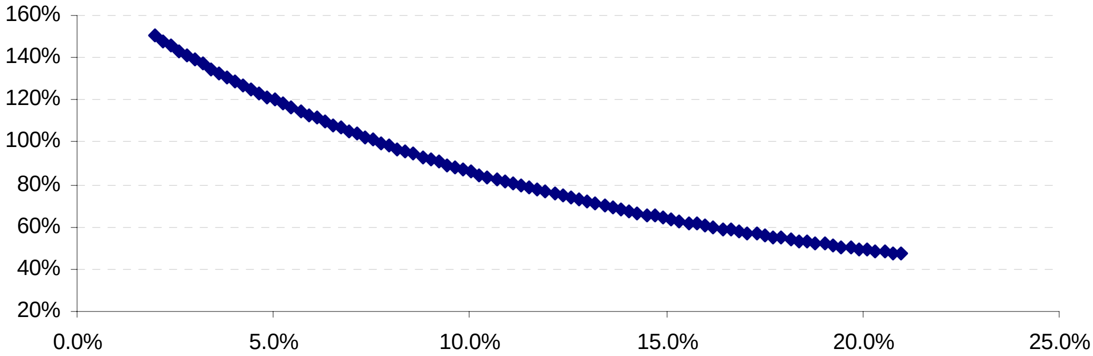
1. If the bond was issued at par, it means that coupon rate (in this case semi-annual) is equal to YTM (on semiannual basis). As semiannual coupon rate is 3.775% (=7.55/2). So the annual YTM (bond equivalent yield) will be:

$$3.775\% \cdot 2 = 7.55\%$$

2. As we can see from the graph[1] the price for bond is a decreasing and convex function in discount rate – it means that absolute value of duration is also decreasing as discount rate is increasing.

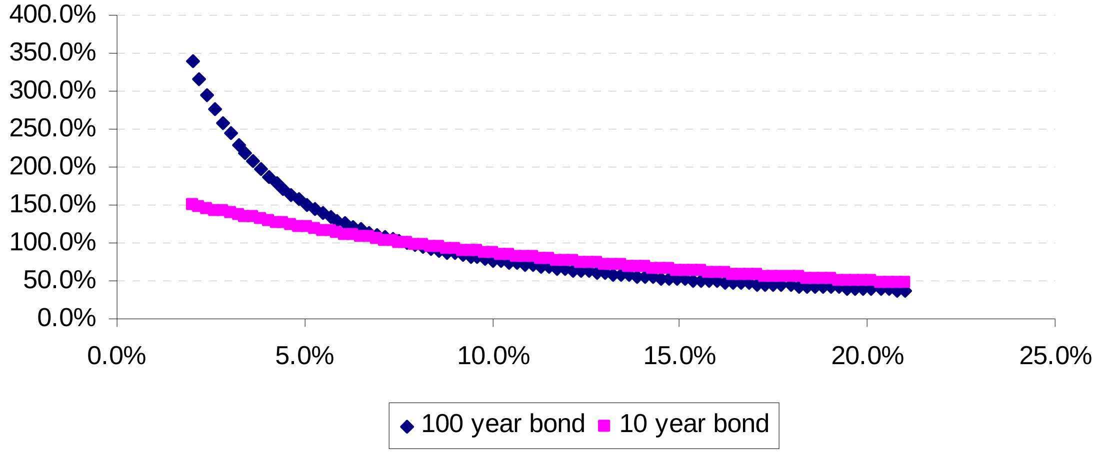


3. Below one can find a graph for the 10 year bond price vs discount rate. The price is also a decreasing and convex function of discount rate. But the slope of the curve (and duration) and convexity is significantly different from the 100 year bond (see next graph).



[1] Discount rate are presented as effective annual rates. Graph plots Price vs discount rate.

From the graph we can find out, that 100 year bond is more sensitive to changes in discount rates than 10 year bond. In addition, 100 year bond's price function is more convex than one of the 10 year bond.



4. As duration is an increasing function of maturity of a bond, 100 year bond should have a higher duration than 10 year bond. Using 8% YTM (or 4% semiannual YTM), 100 year bond has a duration of 25,99 (semesters), while 10 years bond has duration of only 14,29 (semesters).

5. If a bond equivalent yield will increase by 1% that implies that semiannual YTM will increase by 0.5%: from 4% to 4.5%. In this case price of a 100 and a 10 year bond will be 83.9% and 90.6% (or a decrease of -11% and -6%). Using duration formula would lead to an overestimation of price decline, i.e. duration will predict more rapid price decline. This happens because duration formula assumes a constancy of duration. However, as yields increases (for these bonds) the duration (absolute value) will decrease. This, in turn, follows from a property of price function, which is convex. In other words, duration is a first order approximization of price function, that does ignore second order effects (i.e. change in slope of price function) and other higher order components. The relative mistake for 100 year bond is significantly larger fro 10 year bond, due to the ignorance of convexity.

Method	Maturity, years	Price at YTM=4.5%	Change in price, %
Exact price	100	83.9%	-11.1%
	10	90.6%	-6.6%
Price based on Duration	100		-12.5%
	10		-6.9%

6. 100 year bond is more sensitive to interest rate fluctuations than 10 years bonds, because it has much higher duration. There is also an intuitively appealing explanation — 100 year bond at least has higher interest rate change risk: buying it implies that buyer has a fixed stream of cash flows. If suddenly interest rates are growing than price

of this bond falls and buyer cannot sell a bond with profit. The only way to get YTM is to way 100 years when the principal will be repaid — but that's too long.

7.

a. Call feature implies that company, which had issued a bond, can buy it back at certain pre-specified conditions and at certain time (interval). In fact this feature is an in-build option.

b. This bond has an option which should be priced. This option gives additional flexibility to WD and reduced interest rate change risk. From other hand, this risk reduction is shifted towards buyers of this bond. The extent to which this option will influence the price depends upon the terms of issuance and investors' expectation about future interest rates. For the past 20 years interest rates have declining in the world, so the investors might have beliefs that inflation will be low ("anchored expectations") and so the interest rates. In this case, 7.55% will be a really nice investment (ceteris paribus) in 10-20-30 years.

   In fact, WD YTM was 7.55% will call feature, while Coca-Cola issued very much the same bond but without an option at 7.455%. The difference is not huge: the reasons for this might be very different — WD was the first to issue such a long maturity bond, the demand was extremely high; the investors were not prepared to make predictions for 100 years ahead.

c. But even if investors does not price loose of flexibility, it makes a lot of sense for WD — after 30 years, interest rates might be as low as 2-3%. If the company did not have a call option it will have to pay 7.55% versus 2-3% market rates. In this situation, issuing a 100 year bond will lead to a transfer of wealth from shareholders to debtors in future.

8.

a. Given expectation of rising interest rates, it does not matter which bond to buy given that their prices (SByes and SBno) are equal. If the interest rates go up WD does not have any incentives to call back bonds. If the price of a callable bonds is significantly lower than price of SBno, than I will buy SByes given my expectations. It will be the same bond as SBno, because WD has no incentive to call it back.

b. Higher uncertainly should affect price of SByes more than price of SBno. So, the price of SByes should change more than the price of SBno. However, it's hard to make a final answer without specifying a correct option model here.

9.

a.
$$f_{2009} = \frac{(1+s_{2009})^4}{(1+s_{2008})^3} - 1 = 7.0\%$$

$$f_{2010} = \frac{(1+s_{2010})^5}{(1+s_{2009})^4} - 1 = 8.0\%$$

$$f_{2011} = 7.2\%$$

$$f_{2012} = 7.6\%$$

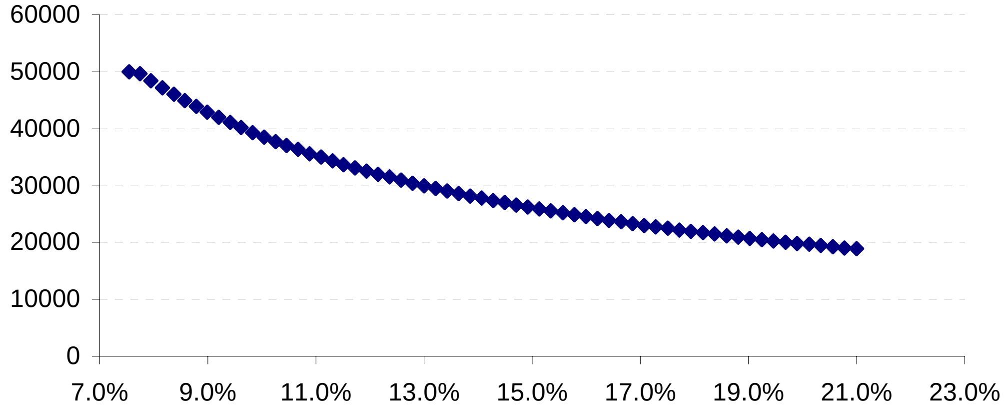
$$f_{2013} = 7.2\%$$

b. Given the yield curve we can use spot rates to discount the corresponding cash flow. After the 2013 we assume rate to be constant to 6.5% ($r = 6.5\%$):

$$P=\frac{C}{1+s_{2006}}+\frac{C}{(1+s_{2007})^2}+\frac{C}{(1+s_{2008})^3}+\ldots+\frac{C}{(1+s_{2013})^8}+\sum_{t=9}^{88}\frac{C}{(1+r)^t}+\frac{FV}{(1+r)^{88}}=118.3\%$$

10.

If interest rates will be lower than 7.55% the company will call bond back at 103.2% of it face value or 50000*103.02%=51510[2]. Otherwise, the price of a bond will be always lower than 25.000 (see graph). At the rate of 7.55% WD is indifferent between calling and not calling bond back. We assume that it will not call it back. Than the value of a position will be 50000 (100% price). With higher rates the value of a position is shown in the graph below.



To sum up, if the rate is lower 7.55% we do not have a position in WD's bond but only "money in the pocket" of 51510. If the rates are higher 7.55 the value of the position in WD's bond is shown in the graph and will be lower than 50000.

[2] We ignore transaction cost and (s,S) considerations, assuming that WD will call back the bond if interest is lower than 7.55%

B. CFA questions (6th edition)

Question 7 (Ch.14)

a.

i. Current yield $= \dfrac{70}{960} = 7.29\%$

ii. Yield to maturity $\approx$ 8%

iii. Realized compounded rate = 8.498%

time	payment	FV
0.5	35.00	40.49
1	35.00	39.33
1.5	35.00	38.20
2	35.00	37.10
2.5	35.00	36.03
3	1 035.00	1 035.00
Sum of FV		1 226.15
PV		960.00

b. Shortcomings:

i. Current yield – does not represent the return that investors get, ignores the redistribution of coupons payments in time (e.g., investors prefers 8% coupons paid semi-annually rather than 8% annually)

ii. YTM – return that the investor will get if he keeps bonds until the maturity and reinvest coupons at the same as rate as YTM (or buys more of this bond).

iii. Realized compound yield - depends on market performance and your strategy. Hardly predictable.

Question 31 (Ch.14)

a.3

b.3

c.2

d.3

e.2

f.3

Question 15 (Ch.15)

Zero coupon US Treasuries can have a higher yield because of the interest rate risk. There is always a risk of growing (changing) rates in future. Coupon bond has low interest rate risk (duration) because it provides coupon payments that partially offset price decline.

Question 20 (Ch.15)

a. We can solve for 5 year spot rate given data — it will be 7.13:

Year	Payment	Spot rate	PV
1	70	5.0%	66
2	70	5.21%	63
3	70	6.05%	59
4	70	7.16%	53
5	1070	7.13%	758
			Price 1000

Then the forward price can be easily calculated as:

$$f_5 = \frac{(1+s_5)^5}{(1+s_4)^4} - 1 = 7.0\%$$

i. YTM – IRR that equals the present value of a bond's cash flows to price. The investor gets this yield only if he keeps the bond up to its maturity and reinvests coupons at the same rate.

ii. Spot rate – YTM on zero-coupon bonds.

iii. Forward rate – implied interest rate that shows an interest rate for a specific time period. E.g., from t-1 to t.

YTM is a IRR that equals PV of payments with price. Spot rates are used to discount the corresponding cash flows of a bond. In fact, YTM is an average spot rate which is used to get a bond yield. So any movement in spot rate will be reflected in YTM. Forward rate can be "extracted" from the spot rates. It shows an interest rate between periods of time, so it's directly linked with the spot rates (yield curve). That direct link between spot and forward rates implies the same link between forward rates and YTM (as with spot rates).

Question 25 (ch.15)

a. i. The annualized forward rate for 2 years (years 4-5) is 6.07%

ii. The expectations hypothesis states that the forward rate equals the market expectation of the future short interest rate, so there is no arbitrage opportunity. It implies possibility to "extract" forward rates spot rates.

b. Price = 987.09711

CF	Discount factor
90	0.8849558
90	0.7971939
90	0.7311914
90	0.6830135
1090	0.6499314
Price	987.0

Question 8 (Ch.16)

a. 4

b. 2

c. 1

d. 1

e. 3

f. 1

g. 1

h. 3

Question 10 (Ch.16)

a. Modified duration $= \dfrac{10}{1.08} = 9.26$ years

b. At the time moment equal to duration the bond price is almost non sensitive to interest movements. Maturity itself doesn't possess a lot of information about sensitivity to interest rates: because there are different factors – distribution of coupons payments, timing of principal repayment, etc. However, ceteris paribus, longer maturity leads to higher duration.

c. If the coupons were 4% instead of 8% the duration would be higher, because PV of coupons payments decrease.

d. If the maturity was 7 years instead of 15 years the duration would be lower (¡ 7).

e. Convexity is the property of price-yield function. It reflect that duration changes with changing yields. Second order approximization looks like:

$$\frac{dP}{P} = -D \cdot dr + \frac{1}{2} \cdot Convexity \cdot \left(dr\right)^2$$

Question 32 (ch.16)

Using Duration only:

Portfolio 1: 0.0075*4.83*0.5 – 0.005*23.81*0.5 =-0.0414125 = -4.14%

Portfolio 2: -0.0025*14.35 = -0.035875 = -3.59%

So the first portfolio seems to be more sensitive to this type of interest changes.

C. Additional exercises

1)

Investor paid: 974.69 or 97.5%

Money investor gets at maturity – price of a bond plus the last dividend payments and reinvestment income:

$$126.1\% = 100\% + 8\% + 8\%(1 + c_2) + 8 * (1 + c_1)(1 + c_2)$$

, where $c_1 = 7.20\%$ — reinvestment rate at the end of the first year, $c_2 = 9.40\%$ — reinvestment rate at the end of the third year.

So, realized annualized return is 9%:

$$\left(\frac{126.1\% - 97.5\%}{97.5}\right)^{1/3} - 1 = 9.0\%$$

2)
Price of a bond with a coupon rate less than YTM has a growing pattern in time, given that YTM is constant: i.e. price of a bond is growing in time at equals it's face value at maturity. Let's show it: P_0 — price the year before, P_1 — price now.

$$P_0 = \frac{6\%}{(1+r)} + \frac{6\%}{(1+r)^2} + \frac{100\%}{(1+r)^2}$$

$$P_1 = \frac{6\%}{(1+r)} + \frac{100\%}{(1+r)}$$

It can be easily shown that: $P_0 = \frac{P_1}{(1+r)} + \frac{6\%}{(1+r)}$ or $P_0(1+r) = P_1 + 6\%$

This implies that $P_0 < P_1$. In our case $P_0 = 96.43\%$, $P_1 = 98.15\%$

3) I propose two solutions depending on the treatment of assumptions in the exercise. In the first one I assume that one year spot rate next year is equal to 2 years spot rate now. In the second version — I assume that one year spot rate next year is equal to forward rate from 1 to 2 (this year).

First version:

This year:

$$s_1 = 5\%, f_{1,2} = 6\%, f_{2,3} = 6.5\%$$
$$s_2 = \sqrt{(1+s_1)(1+f_{1,2})} - 1 = 5.5\%$$
$$s_3 = \sqrt[3]{(1+s_2)^2(1+f_{2,3})} - 1 = 5.83\%$$

So the price of a 3-year zero coupon bond is:

$$P_0 = \frac{100\%}{(1+s_3)^3} = 84.36\%$$

Next year — new spot rates are (with a symbol ~):

$$\tilde{f}_{1,2} = f_{2,3} = 6.5\%$$
$$\tilde{s}_1 = s_2 = 5.5\%$$

So, we can compute new spot rate for second year (from now):

$$\tilde{s}_2 = \sqrt{(1+\tilde{s}_1)(1+\tilde{f}_{1,2})} - 1 = 6.00\%$$

So, the price of a 3 year zero coupon bond after one year with these new rates is:

$$P_1 = \frac{100\%}{(1+\tilde{s}_2)^2} = 89.00\%$$

So, the holding period return is 5.50%:

$$\frac{P_1 - P_0}{P_0} = \frac{84.36\% - 89.00\%}{84.36\%} = 5.50$$

Second version:

Next year — $\tilde{s}_1$ will be equal to second year forward rate ($f_{1,2}$)

$$\tilde{f}_{1,2} = f_{2,3} = 6.5\%$$
$$\tilde{s}_1 = f_{1,2} = 6.0\%$$

Than the price will be:

$$P_1 = \frac{100\%}{(1+\tilde{f}_{1,2})(1+\tilde{s}_1)} = 88.58\%$$

So, the holding period return is 5.0%:

$$\frac{P_1 - P_0}{P_0} = \frac{84.36\% - 88.58\%}{84.36\%} = 5.0\%$$