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Financial Econometrics-II

Amsterdam Business School, UvA
MIF Program

Assignment #2: VaR

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Question 1. Estimating ARMA-GARCH model for a1 and a2

Estimates for a1

We estimate the model using Bollerslev-Wooldrige covariance matrix. We find that GARCH(1,1) model is the most suitable and parsimonious for a1. Below the table you can find specification tests.

Final Specification Dependent Variable: A1

GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.053554	0.026907	1.990346	0.0466
		Variance Equation		
C	0.031123	0.013409	2.321057	0.0203
RESID(-1)^2	0.044077	0.010664	4.133203	0.0000
GARCH(-1)	0.949257	0.011251	84.37329	0.0000

Specification tests:

1) No Asymmetric news curve

Estimating GARCH-T model shows that parameter, which corresponds to nonlinearity of news curve, is not significantly different from zero at 5% level.

2) No autocorrelation at 5%

As one can find in the appendix, there is no autocorrelation in residuals.

3) No ARCH effects

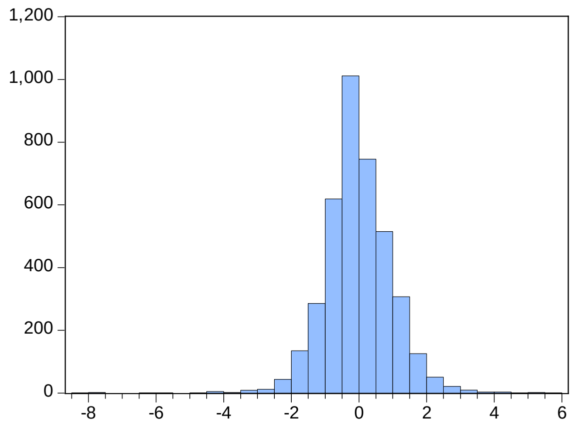
As one can see p-value of F-statistics, which correspond to the null hypothesis of no ARCH effects, is higher than 5% critical level. That implies that we do not reject the null hypothesis at 5% significance level.

Heteroskedasticity Test: ARCH

F-statistic	0.926011	Prob. F(30,3854)	0.5820
Obs*R-squared	27.80336	Prob. Chi-Square(30)	0.5809

4) Nonnormal residuals

According to JB statistics a1's residual are nonnormal, because we reject the null hypothesis of normal distribution. P-value of JB is lower than 5% critical level.



Series: Standardized Residuals	
Sample 3/12/1990 3/11/2005	
Observations 3915	
Mean	-0.014648
Median	-0.032826
Maximum	5.698128
Minimum	-8.365202
Std. Dev.	1.000036
Skewness	-0.302711
Kurtosis	8.185601
Jarque-Bera	4446.297
Probability	0.000000

Estimates for a2

We estimate the model using Bollerslev-Wooldrige covariance matrix. We find that ARMA(1,1)-GARCH(1,1) model is the most suitable and parsimonious for a2. Below the table you can find specification tests.

Dependent Variable: A2
Presample variance: backcast (parameter = 0.7)
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.051074	0.013203	3.868359	0.0001
AR(1)	0.665917	0.059631	11.16736	0.0000
MA(1)	-0.768534	0.051364	-14.96258	0.0000
		Variance Equation		
C	0.019389	0.006070	3.194137	0.0014
RESID(-1)^2	0.048919	0.008284	5.905495	0.0000
GARCH(-1)	0.941188	0.009498	99.09755	0.0000

Sum squared resid	7414.286	Schwarz criterion	3.341337
Log likelihood	-6514.180	Hannan-Quinn criter.	3.335134
F-statistic	14.08061	Durbin-Watson stat	1.964370
Prob(F-statistic)	0.000000		
Inverted AR Roots	.67		
Inverted MA Roots	.77		

Specification tests:

5) No Asymmetric news curve

Estimating GARCH-T model shows that parameter, which corresponds to nonlinearity of news curve, is not significantly different from zero at 5% level.

Variance Equation

C	0.022036	0.006477	3.402120	0.0007
RESID(-1)^2	0.039999	0.008318	4.808652	0.0000
RESID(-1)^2*(RESID(-1)<0)	0.020413	0.013667	1.493543	0.1353
GARCH(-1)	0.938122	0.009794	95.78332	0.0000

6) No autocorrelation at 5%

As one can find in the appendix, there is no autocorrelation in residuals.

7) No ARCH effects

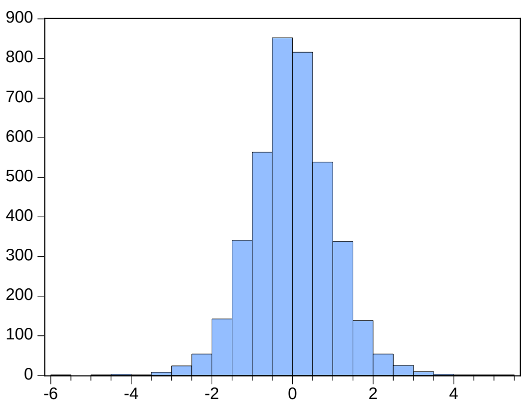
As one can see p-value of F-statistics, which correspond to the null hypothesis of no ARCH effects, is higher than 5% critical level. That implies that we do not reject the null hypothesis at 5% significance level.

Heteroskedasticity Test: ARCH

F-statistic	1.213305	Prob. F(100,3713)	0.0755
Obs*R-squared	120.6872	Prob. Chi-Square(100)	0.0780

Nonnormal residuals

According to JB statistics a2's residual are nonnormal, because we reject the null hypothesis of normal distribution. P-value of JB is lower than 5% critical level.



Series: Standardized Residuals
Sample 3/13/1990 3/11/2005
Observations 3914

Mean	-0.009568
Median	-0.019811
Maximum	5.075962
Minimum	-5.672408
Std. Dev.	0.997459
Skewness	-0.011688
Kurtosis	4.219531
Jarque-Bera	242.6359
Probability	0.000000

Question 2

We use a historical correlation coefficient of 0.24 ($\overline{\rho}$) between a1 and a2. We construct VaR using estimates of variance for a1 and a2 from GARCH models ($\hat{\sigma}_{1t}^2$ *and* $\hat{\sigma}_{2t}^2$):

$$\sigma_{pt}^2 = \hat{\sigma}_{1t}^2(0.5)^2 + \hat{\sigma}_{2t}^2(0.5)^2 + 2\overline{\rho}\hat{\sigma}_{1t}\hat{\sigma}_{2t}(0.5)^2$$

and 1-% VaR:

$$VaR = -2.326 \cdot \sigma_{pt}$$

Riskmetrick VaR is estimated in the same fashion, assuming a correlation coefficient of 0.24 ($\overline{\rho}$) between a1 and a2, and using Riskmetrics © estimates of conditional variances ($\hat{\sigma}_{1t}^2$ *and* $\hat{\sigma}_{2t}^2$):

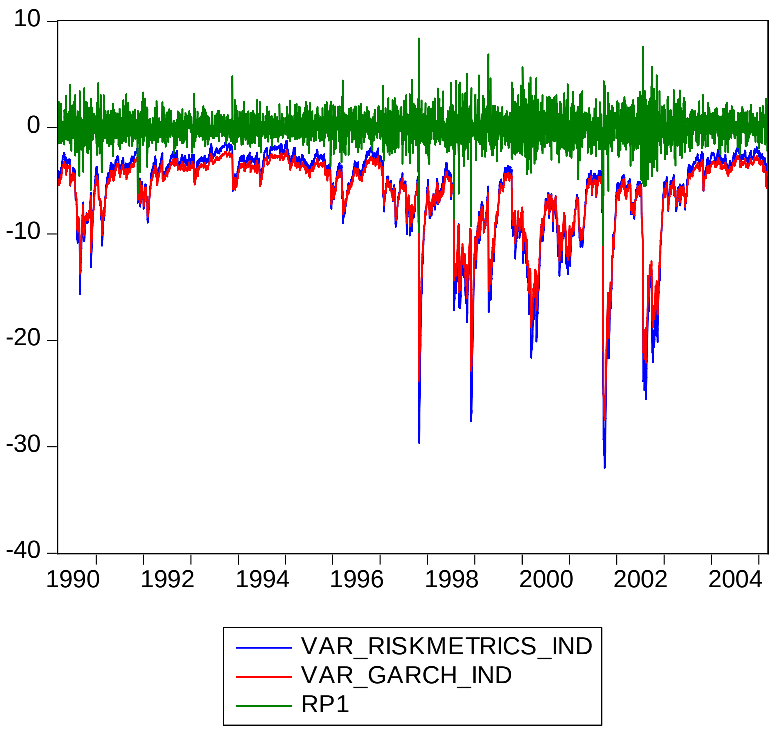
$$\hat{\sigma}_{jt}^2 = (1-\lambda)\sum_{i=0}^{\infty}\lambda^i R_{jt-i}^2, \text{where } \lambda = 0.94 \text{ and } j = 1,2$$

portfolio variance:

$$\sigma_{pt}^2 = \hat{\sigma}_{1t}^2(0.5)^2 + \hat{\sigma}_{2t}^2(0.5)^2 + 2\overline{\rho}\hat{\sigma}_{1t}\hat{\sigma}_{2t}(0.5)^2$$

And 1-% VaR:

$$VaR = -2.326 \cdot \sigma_{pt}$$



Backtesting

We construct hit function, with which we backtest the models. In the table below you can find the statistics for GARCH and Riskmetrics.

$$I_{t+1} = \begin{cases} 1, & \text{if } R_{t+1} < -VaR^{p}_{t+1}, \\ 0, & \text{if } R_{t+1} > -VaR^{p}_{t+1}. \end{cases}$$

Back testing		
	GARCH-IND (GARCH model for a1 and a2 separately)	Riskmetrics-ind (separately for a1 and a2)
Mean (estimate of probability)	0.010475	0.013027
VAR>RP, number of cases	41	51
N	3914	3915

To test a hypothesis: $H_0 : \Pr(R_{t+1} < -VaR_{t+1}) = 0.01$ we use the following statistics:

$$t = \frac{\hat{\pi} - 0.01}{\sqrt{\hat{\pi}(1-\hat{\pi})/T}} \overset{T\to\infty}{\sim} N(0,1)$$

We will assume that T=3914 is enough to use approximation to normal distribution.

	GARCH	RiskMetrics
t-stat	0.29	1.67
$F^{-1}_{0.01}$	2.326	

As both t-stats are lower than 2.326 (critical 1% value based on normal distribution), so we do not reject the null hypothesis of p=1%. That implies that both methods (GARCH and Riskmetrics) on average should lead to p=1%. However, GARCH is more precise than Riskmetrics as it has less observation VAR>RP (41 vs 51).

Question 3

We estimate the model using Bollerslev-Wooldrige covariance matrix. Final Specification — GARCH (1,1) with MA(2) in returns equation.

Dependent Variable: RP1
Method: ML - ARCH (Marquardt) - Normal distribution
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.057663	0.017831	3.233899	0.0012
MA(2)	-0.046561	0.017951	-2.593831	0.0095
	Variance Equation			
C	0.015037	0.005722	2.628014	0.0086
RESID(-1)^2	0.045135	0.009328	4.838654	0.0000
GARCH(-1)	0.947183	0.009297	101.8794	0.0000

R-squared	0.002437	Mean dependent var	0.033156
Adjusted R-squared	0.001416	S.D. dependent var	1.328782
S.E. of regression	1.327840	Akaike info criterion	3.258846
Sum squared resid	6893.956	Schwarz criterion	3.266857
Log likelihood	-6374.192	Hannan-Quinn criter.	3.261689
F-statistic	2.387902	Durbin-Watson stat	2.029178
Prob(F-statistic)	0.048883		

Inverted MA Roots	.22	-.22

Specification tests:

1) No Asymmetric news curve

Estimating GARCH-T model shows that parameter, which corresponds to nonlinearity of news curve, is not significantly different from zero at 5% level.

C	0.018518	0.005993	3.089631	0.0020
RESID(-1)^2	0.027241	0.009799	2.780057	0.0054
RESID(-1)^2*(RESID(-1)<0)	0.036260	0.019890	1.823038	0.0683
GARCH(-1)	0.944543	0.008800	107.3291	0.0000

2) No autocorrelation at 5%

Originally, with GARCH(1,1) there is an autocorrelation in the second lag (appendix). So we try both AR(2) and MA(2), which solve this problem. As Schwarz criteria shows MA(2) is more preferable. As we add MA(2) one can find in the appendix, there is no autocorrelation in residuals.

3) No ARCH effects

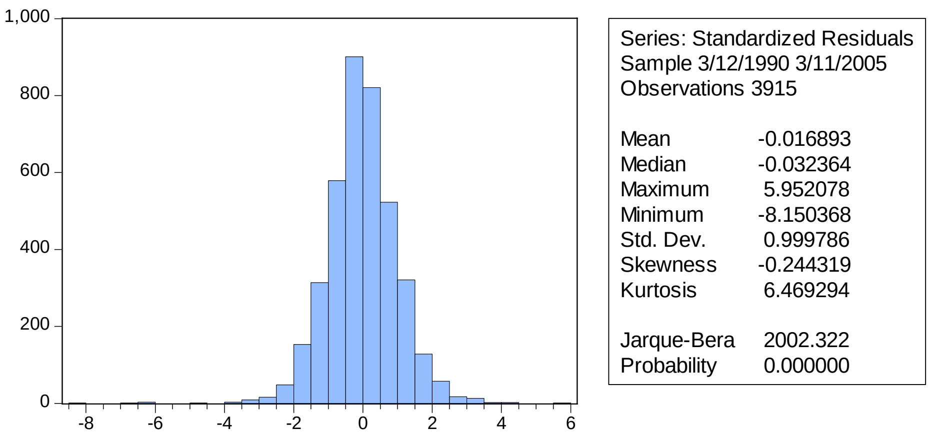
As one can see p-value of F-statistics, which correspond to the null hypothesis of no ARCH effects, is higher than 5% critical level. That implies that we cannot reject the null hypothesis at 5% significance level.

Heteroskedasticity Test: ARCH

F-statistic	0.929249	Prob. F(30,3854)	0.5769
Obs*R-squared	27.89990	Prob. Chi-Square(30)	0.5757

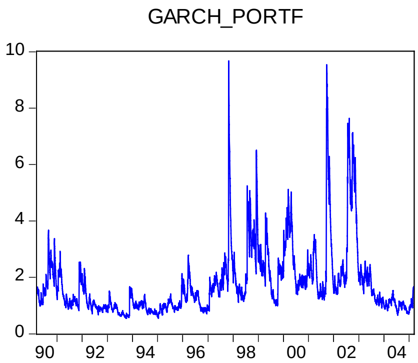
4) Nonnormal residuals

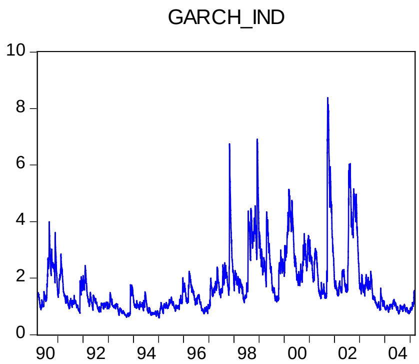
According to JB statistics, we reject the null hypothesis of normality (at 5% level) as p-value of JB stat. is lower than 5% level.

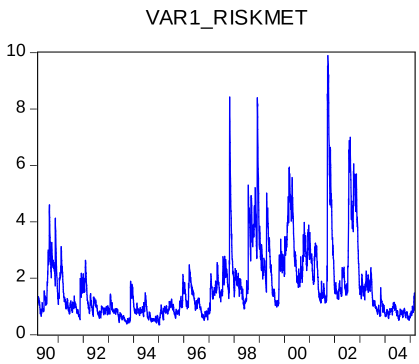


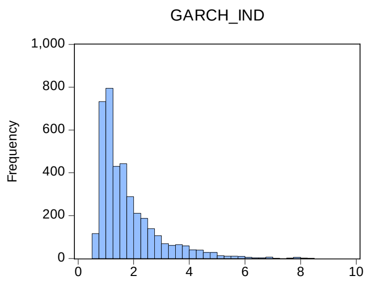
Below you can find conditional variance graph for three methods. As you can find, they yield very common results, which might be a sign of a validity of constant correlation assumption. Because we obtain the same pattern for models that assume constant correlation (GARCH_IND and RISK_MET) and those that does not (GARCH_PORTF).

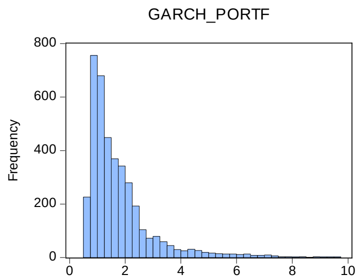
GARCH-IND and VAR1-RISKMET – from question 2, GARCH-portf is based on a3 returns.

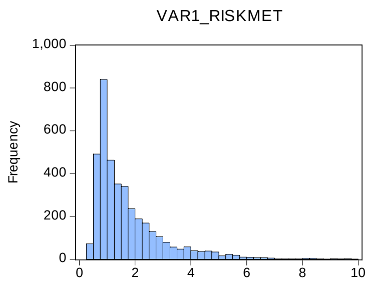










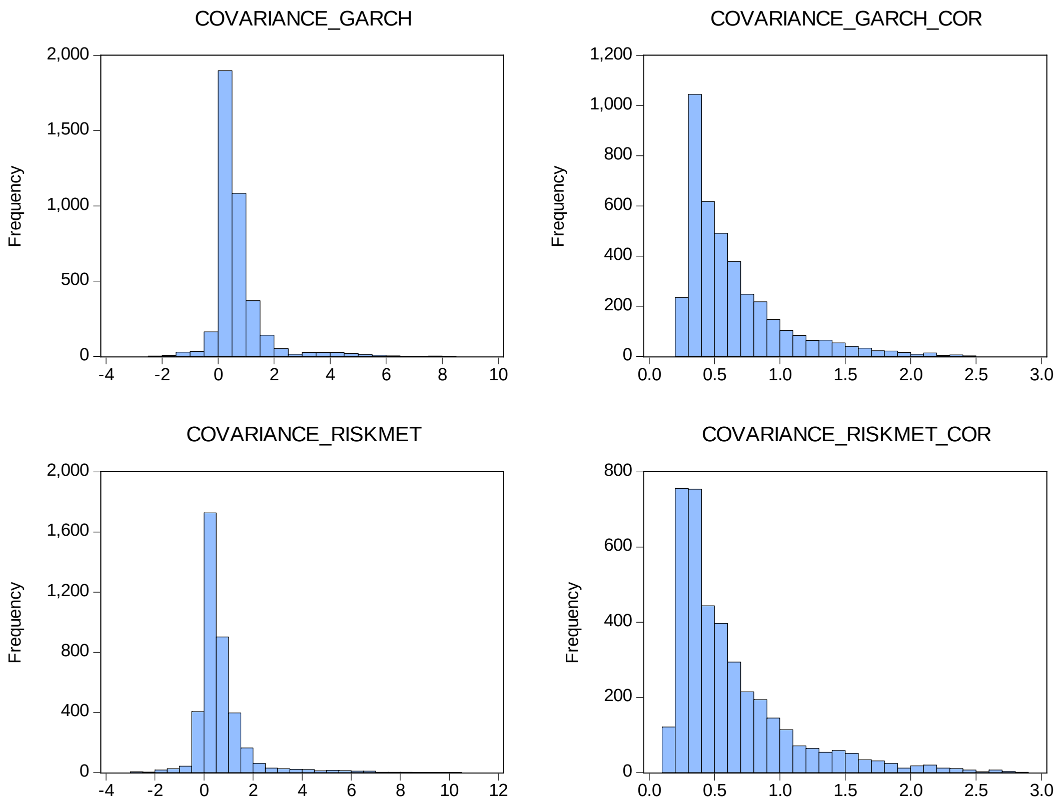


Question 4

From your viewpoint is there any problem with this estimator? There might be several problems:

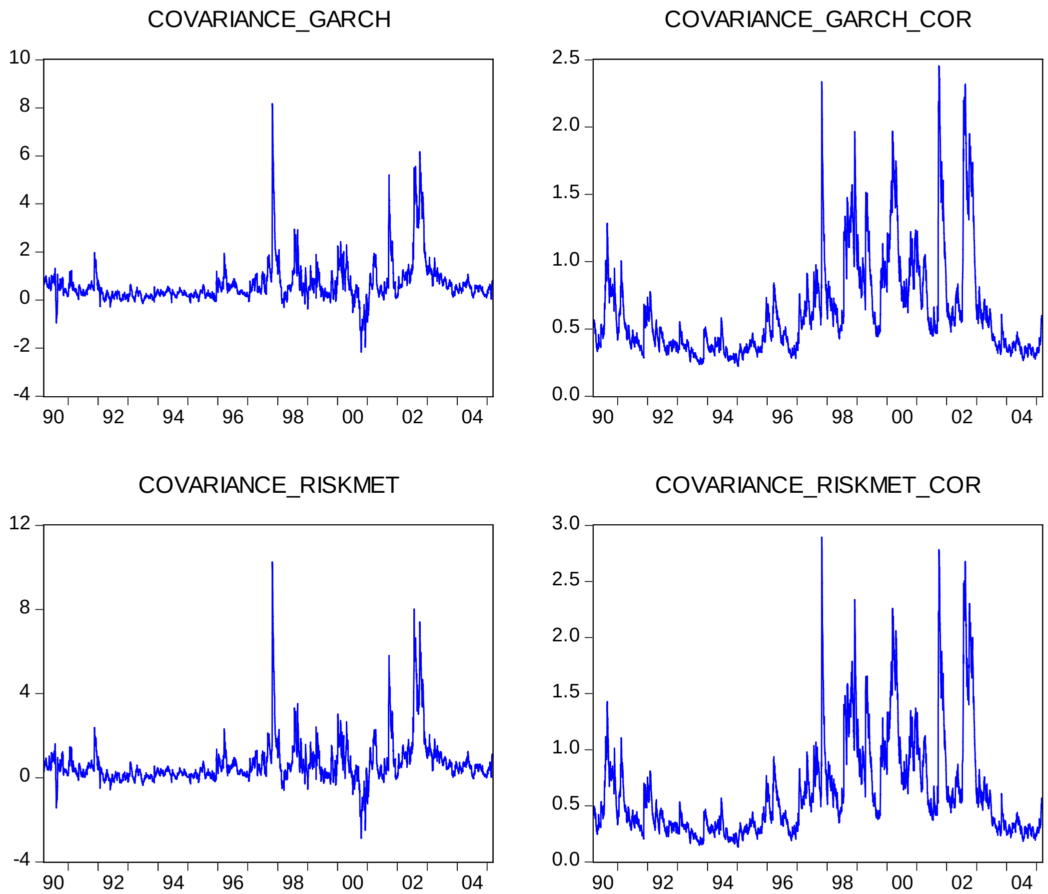
- Precision. We add together three estimates from three different GARCH models, which use the same information. This may lead to a high correlation between estimation errors of variance, so when we add them together they will not diversify fully
- Correlation estimation. There is no guarantee that correlation coefficient estimated with this covariance will lie in -1 to 1 interval.
- Specification problems. If we misspecify GARCH model for conditional volatility, that will distort results.

Below you can find graphs with two kind of covariances – based on formula in question 4 (GARCH and Riskmetrics) and constant conditional correlation.[1]

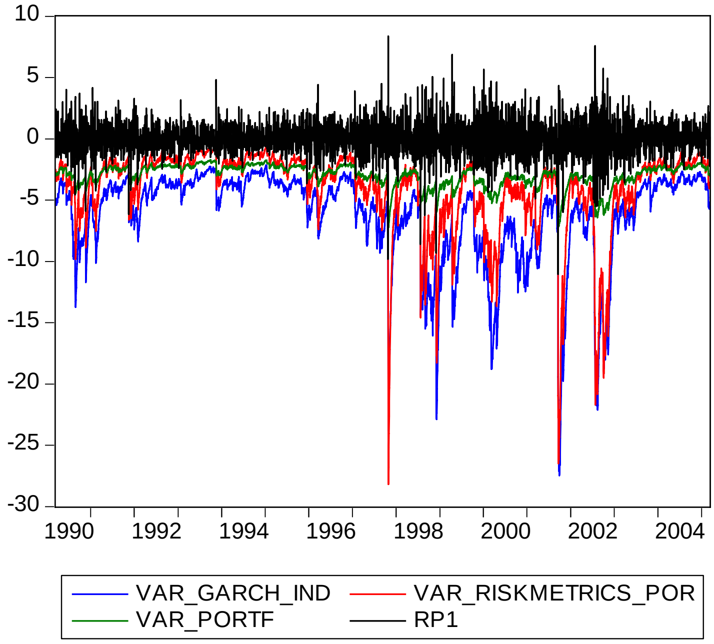


[1] Comment to graph -COVARIANCE-GARCH (GARCH for question 4), COVARIANCE_GARCH_COR(GARCH for a1 and a2 with constant correlation), COVARIANCE_RISKMET – Riskmetrics estimates of portfolio's VaR based on a3 (for question 4 equation), COVARIANCE_RISKMETRICS_COR – Riskmetrics with constant correlation.

As one can notice: distribution patterns are quite different between two approaches (formulae in question 4 and constant correlation) — first approach gives more volatile covariance estimates and allows for negative covariance. From distribution graph it is quite clear that the first approach leads to more "heavy tails", which seem to be intuitive: in a crisis time correlation and volatility can increase simultaneously, leading to a higher covariance compared to constant correlation model. The constant probability approach does not count this fact, what might lead to underestimation of VaR and wrong asset allocation decisions in "bad" times. So, here it becomes obvious that constant correlation model leads to different results when the first approach (formulae in quest.4). As we have shown in question 3, conditional variance with GARCH applied to portfolio returns leads to the same result as with those obtained with an assumption of constant correlation. So it might be possible that difference between constant correlation approach and the first approach (formulae in quest.4) is mostly due to unreliability of the latter (because of possible problems outlined in the beginning of this question).



Question 5



Back testing				
	GARCH-IND (GARCH model for a1 and a2 separately)	GARCH for Portfolio	Riskmetrics for portfolio	Riskmetrics-ind (separately for a1 and a2)
Mean	0.010475	0.009962	0.012005	0.013027
VAR>RP	41	39	47	51
N	3914	3914	3915	3915

To test a hypothesis: $H_0 : \Pr(R_{t+1} < -VaR_{t+1}) = 0.01$ we use the following statistics:

$$t = \frac{\hat{\pi} - 0.01}{\sqrt{\hat{\pi}(1-\hat{\pi})/T}} \overset{T\to\infty}{\sim} N(0,1)$$

We will assume that T=3914 is enough to use approximation to normal distribution.

Back testing				
	GARCH-IND (GARCH model for a1 and a2 separately)	GARCH for Portfolio	Riskmetrics for portfolio	Riskmetrics-ind (separately for a1 and a2)
t-stat	0.29	-0.024	1.15	1.67
$F_{0.01}^{-1}$	2.326			

As all t-stats are lower than 2.326 (critical 1% value based on normal distribution), so we do not reject the null hypothesis of p=1% in all cases. That implies that all methods (GARCH and Riskmetrics) on average should lead to p=1%. However, GARCH for portfolio (when it is estimated for a3 directly) is more precise than all other methods, as it has only 39 exceptions, with estimate of probability VAR>RP less than 1%.

Appendix

Autocorrelation for residual in GARCH (1,1) for a1.

Short version – 20 lags.

Date: 03/07/08 Time: 23:05
Sample: 3/12/1990 3/11/2005
Included observations: 3915

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
| |	| |	1	0.017	0.017	1.1444	0.285
| |	| |	2	-0.032	-0.033	5.2339	0.073
| |	| |	3	0.003	0.004	5.2650	0.153
| |	| |	4	-0.026	-0.028	7.9887	0.092
| |	| |	5	-0.003	-0.002	8.0269	0.155
| |	| |	6	0.009	0.007	8.3191	0.216
| |	| |	7	0.003	0.003	8.3552	0.302
| |	| |	8	0.004	0.004	8.4302	0.393
| |	| |	9	0.010	0.010	8.8277	0.453
| |	| |	10	0.037	0.037	14.106	0.168
| |	| |	11	-0.006	-0.007	14.271	0.218
| |	| |	12	0.007	0.010	14.489	0.271
| |	| |	13	0.004	0.004	14.560	0.336
| |	| |	14	-0.011	-0.009	15.045	0.375
| |	| |	15	0.004	0.004	15.103	0.444
| |	| |	16	-0.011	-0.012	15.592	0.482
| |	| |	17	-0.019	-0.018	16.939	0.458
| |	| |	18	-0.020	-0.021	18.527	0.421
| |	| |	19	0.013	0.012	19.163	0.446
| |	| |	20	0.008	0.004	19.401	0.496

Autocorrelation for residual in ARMA(1,1)-GARCH (1,1) for a2.

Short version – 20 lags.

Date: 03/07/08 Time: 23:13
Sample: 3/13/1990 3/11/2005
Included observations: 3914
Q-statistic probabilities adjusted for 2 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
| |	| |	1	0.025	0.025	2.4754	
| |	| |	2	-0.001	-0.002	2.4803	
| |	| |	3	0.014	0.014	3.2426	0.072
| |	| |	4	-0.005	-0.006	3.3556	0.187
| |	| |	5	-0.002	-0.001	3.3662	0.339
| |	| |	6	0.008	0.008	3.6245	0.459
| |	| |	7	0.009	0.009	3.9341	0.559
| |	| |	8	-0.003	-0.004	3.9782	0.680
| |	| |	9	0.015	0.015	4.8235	0.681
| |	| |	10	0.012	0.011	5.4200	0.712
| |	| |	11	-0.015	-0.015	6.2967	0.710
| |	| |	12	0.004	0.004	6.3510	0.785
| |	| |	13	-0.025	-0.026	8.8792	0.633
| |	| |	14	0.006	0.008	9.0193	0.701
| |	| |	15	0.018	0.017	10.273	0.671
| |	| |	16	0.018	0.018	11.618	0.637
| |	| |	17	-0.007	-0.008	11.810	0.693
| |	| |	18	0.002	0.002	11.825	0.756
| |	| |	19	-0.007	-0.008	12.018	0.799
| |	| |	20	-0.016	-0.015	13.038	0.789

Autocorrelation for residual in ARMA(0,2)-GARCH (1,1) for portfolio returns.

Short version – 20 lags.

Date: 03/07/08 Time: 23:46
Sample: 3/12/1990 3/11/2005
Included observations: 3915
Q-statistic probabilities adjusted for 1 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
| |	| |	1	-0.016	-0.016	1.0021	
| |	| |	2	-0.001	-0.001	1.0049	0.316
| |	| |	3	-0.006	-0.006	1.1387	0.566
| |	| |	4	-0.020	-0.021	2.7741	0.428
| |	| |	5	-0.010	-0.011	3.1906	0.526
| |	| |	6	0.009	0.008	3.4868	0.625
| |	| |	7	0.002	0.002	3.5062	0.743
| |	| |	8	-0.002	-0.002	3.5216	0.833
| |	| |	9	0.026	0.025	6.1100	0.635
| |	| |	10	0.041	0.042	12.570	0.183
| |	| |	11	-0.012	-0.011	13.182	0.214
| |	| |	12	0.005	0.004	13.263	0.276
| |	| |	13	0.007	0.009	13.476	0.335
| |	| |	14	-0.018	-0.016	14.777	0.321
| |	| |	15	0.003	0.002	14.809	0.391
| |	| |	16	-0.005	-0.006	14.924	0.457
| |	| |	17	-0.023	-0.023	17.060	0.382
| |	| |	18	-0.026	-0.028	19.666	0.292
| |	| |	19	0.011	0.007	20.135	0.325
| |	| |	20	0.013	0.012	20.763	0.350

GARCH(1,1) for portfolio (a3) returns

Date: 03/03/08 Time: 19:51
Sample: 3/12/1990 3/11/2005
Included observations: 3915

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
| |	| |	1	-0.015	-0.015	0.8786	0.349
| |	| |	2	-0.043	-0.043	8.1300	0.017
| |	| |	3	-0.005	-0.006	8.2156	0.042
| |	| |	4	-0.021	-0.023	9.9148	0.042
| |	| |	5	-0.010	-0.011	10.304	0.067
| |	| |	6	0.009	0.007	10.649	0.100
| |	| |	7	0.002	0.001	10.661	0.154
| |	| |	8	-0.004	-0.004	10.721	0.218
| |	| |	9	0.026	0.026	13.365	0.147
| |	| |	10	0.040	0.041	19.738	0.032
| |	| |	11	-0.014	-0.010	20.490	0.039
| |	| |	12	0.003	0.007	20.538	0.058
| |	| |	13	0.008	0.008	20.760	0.078
| |	| |	14	-0.018	-0.015	22.030	0.078
| |	| |	15	0.004	0.004	22.079	0.106
| |	| |	16	-0.003	-0.006	22.127	0.139
| |	| |	17	-0.024	-0.024	24.391	0.109
| |	| |	18	-0.026	-0.029	27.090	0.077
| |	| |	19	0.012	0.006	27.627	0.091
| |	| |	20	0.013	0.010	28.341	0.102
| |	| |	21	0.007	0.007	28.511	0.126
| |	| |	22	0.013	0.012	29.190	0.140
| |	| |	23	-0.000	0.002	29.190	0.174
| |	| |	24	0.007	0.011	29.392	0.206
| |	| |	25	0.008	0.009	29.672	0.237